SOLVING *Quadratics* BY SQUARE ROOTS

Quadratic Equations of the form $ax^2 + c = 0$

can be solved using the **Square Root Property:**

If $\sqrt{x^2} = \sqrt{c}$, then $x = \pm\sqrt{c}$

① **ISOLATE X^2**

② Take the **SQUARE ROOT** of both sides.

③ **SIMPLIFY** the radical. Place "±" to indicate both answers.

Directions: Solve each equation using the square roots method.

1. $x^2 - 64 = 0$

$+64 \quad +64$

$\sqrt{x^2} = \sqrt{64}$

$x = \pm 8$

2. $7x^2 + 8 = 15$

$-8 \quad -8$

$\frac{7x^2}{7} = \frac{7}{7}$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

3. $81x^2 + 5 = 21$

$-5 \quad -5$

$\frac{81x^2}{81} = \frac{16}{81}$

$\sqrt{x^2} = \sqrt{\frac{16}{81}}$

$x = \pm \frac{4}{9}$

4. $8x^2 + 1 = 17$

$-1 \quad -1$

$\frac{8x^2}{8} = \frac{16}{8}$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

5. $2x^2 - 9 = 55$

$+9 \quad +9$

$\frac{2x^2}{2} = \frac{64}{2}$

$\sqrt{x^2} = \sqrt{32}$

$x = \pm\sqrt{32} = \pm\sqrt{16 \cdot 2}$

$x = \pm 4\sqrt{2}$

6. $9x^2 + 3 = 111$

7. $4 - 3x^2 = -77$

8. $5x^2 + 10 = 310$

9. $-\frac{1}{2}x^2 + 1 = -39$

Simplifying Negative Square Roots

Step 1: Rewrite $\sqrt{-a}$ as $\sqrt{-1 \cdot a}$

Step 2: Break a down if it is not a perfect square.

Step 3: Simplify the radical, recalling that $\sqrt{-1} = i$.

1. $\sqrt{-9}$

$3i$

2. $\sqrt{-196}$

3. $\sqrt{-5}$

4. $\sqrt{-80}$ $= \sqrt{-1 \cdot 20 \cdot 4}$

$= \sqrt{\boxed{-1}\, 5 \cdot \boxed{4 \cdot 4}}$

$\to i \qquad \to 4$

$\boxed{= \pm 4i\sqrt{5}}$

5. $\sqrt{-32}$

6. $\sqrt{-192}$

Solving Equations

7. $x^2 + 81 = 0$

$-81 \quad -81$

$\sqrt{x^2} = \sqrt{-81}$

$\boxed{x = \pm 9i}$

8. $2x^2 + 9 = 1$

$-9 \quad -9$

$\frac{2x^2}{2} = \frac{-8}{2}$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$

9. $4x^2 + 15 = -9$

$-15 \quad -15$

$\frac{4x^2}{4} = \frac{-24}{4}$

$\sqrt{x^2} = \sqrt{-6}$

$\boxed{x = \pm i\sqrt{6}}$

10. $x^2 + 13 = 1$

$-13 \quad -13$

$\sqrt{x^2} = \sqrt{-12}$

$x = \pm i\sqrt{12} = \pm i\sqrt{4 \cdot 3}$

$\boxed{x = \pm 2i\sqrt{3}}$

COMPLETING THE SQUARE

It is possible to take any quadratic equation, create a perfect square trinomial, and solve it in a similar way. This method is called **completing the square.**

① **REWRITE** as $ax^2 + bx = c$

② **DIVIDE** both sides by "a" so it becomes $x^2 + bx = c$

③ **COMPLETE THE SQUARE** by taking half of b, square it, and **ADD IT TO BOTH SIDES** of the equation.

④ **FACTOR** the perfect square trinomial.

⑤ Take the **SQUARE ROOT** of both sides. This will create two cases because a square root has both a positive and negative value.

⑥ **SOLVE** both equations. **SIMPLIFY** all irrational and complex answers.

Directions: Solve each quadratic equation below by completing the square.

3. $x^2 - 18x + 56 = 0$

$-56 \quad -56$

$x^2 - 18x = -56$

$\underbrace{x^2 - 18x + 81} = -56 + 81$

$(x-9)(x-9) = 25$

$\sqrt{(x-9)^2} = \sqrt{25}$

$x - 9 = \pm 5$

$+9 \quad +9$

$x = 5 + 9 = \boxed{14}$

$x = -5 + 9 = \boxed{4}$

$\frac{-18}{2} =$

-9

$(-9)^2 =$

81

4. $2x^2 - 16x = -30$

$\frac{2x^2 - 16x}{2} = \frac{-30}{2}$

$x^2 - 8x = -15$

$\underbrace{x^2 - 8x + 16} = -15 + 16$

$(x-4)(x-4) = 1$

$\sqrt{(x-4)^2} = \sqrt{1}$

$x - 4 = \pm 1$

$+4 \quad +4$

$x = 1 + 4 = \boxed{5}$

$x = -1 + 4 = \boxed{3}$

$\frac{-8}{2} =$

-4

$(-4)^2 =$

16

5. $4x^2 - 8x = -3$

$\frac{4x^2-8x}{4} = \frac{-3}{4}$

$x^2 - 2x = \frac{-3}{4}$

$\frac{-2}{2} =$

-1

$(-1)^2 =$

1

$x^2 - 2x + 1 = \frac{-3}{4} + 1$

$(x-1)(x-1) = \frac{1}{4}$

$\sqrt{(x-1)^2} = \sqrt{\frac{1}{4}}$

$x - 1 = \pm\frac{1}{2}$

$+1 \quad +1$

$x = \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$

$x = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$

6. $3x^2 + 10x + 8 = 0$

7. $x^2 + 16x - 21 = -5$

8. $3x^2 - 30x = 69$

9. $x^2 + 12x + 43 = 0$

10. $4x^2 + 76 = 16x$

STANDARD FORM

$f(x) = ax^2 + bx + c$

⬇

VERTEX FORM

$f(x) = a(x-h)^2 + k$

If the standard form equation is a perfect square (like ex. 1), simply factor to convert to vertex form. If not, you can use a process called **completing the square** to write the equation in vertex form.

① **GROUP** $(ax^2 + bx)$

② If $a \neq 1$, **FACTOR** it outside so it becomes $a(x^2 + bx)$

③ **COMPLETE THE SQUARE** by taking half of b and squaring it. This is the new "c". Add this to to "x2 + bx". This is the perfect square trinomial.

④ **SUBTRACT** $a \cdot c$ from the end of the equation.

⑤ **FACTOR** the trinomial and **SIMPLIFY** the end of the equation.

Directions: Write each function in vetex form. Give the vertex.

1. $f(x) = x^2 + 4x + 4$ Perfect □ !

$(x+2)(x+2)$

$f(x) = (x+2)^2$ ✓

2. $f(x) = x^2 - 8x + 18$

$x^2 - 8x + 18 = 0$

$-18 \quad -18$

$x^2 - 8x = -18$

$x^2 - 8x + 16 = -18 + 16$

$(x-4)^2 = -2$

$f(x) = (x-4)^2 + 2$

$\frac{-8}{2} =$

-4

$(-4)^2$

16

3. $f(x) = -x^2 + 6x - 5$

$-x^2 + 6x - 5 = 0$

$+5 \quad +5$

$\frac{-x^2 + 6x}{-1} = \frac{5}{-1}$

$x^2 - 6x = -5$

$\underbrace{x^2 - 6x + 9} = -5 + 9$

$(x-3)^2 = 4$

$f(x) = (x-3)^2 - 4$

$\frac{-6}{2} =$

$\bigcirc\!\!\!\!-3$

$(-3)^2 =$

9

4. $f(x) = 2x^2 - 12x + 13$